

Structure 1.3.1

Emission Spectra & the Hydrogen Atom

Complete revision notes, worked examples, practice questions & full answers

Covers: Structure 1.3.1, 1.3.2, 1.3.3 (SL + HL) and the spectroscopic part of Structure 1.3.6 (HL only)

Level coding used throughout: SL + HL = examinable for everyone | HL ONLY = AHL, not required at SL

0 How to use these notes

Every section carries a level badge. If you are an **SL student you may skip every pink HL-only block** — nothing later in the notes depends on it. HL students must know *all* of it, because Paper 1B and Paper 2 questions routinely combine the SL description of a spectrum with the HL calculation from the convergence limit.

IB syllabus point	Level	What you must be able to do	Section here
Structure 1.3.1	SL + HL	Qualitatively describe the relationship between colour, wavelength, frequency and energy across the electromagnetic spectrum. Distinguish between a continuous and a line spectrum.	1, 2
Structure 1.3.2	SL + HL	Describe the emission spectrum of the hydrogen atom, including the relationships between the lines and energy transitions to the first, second and third energy levels.	3, 4
Structure 1.3.3	SL + HL	Deduce the maximum number of electrons that can occupy each energy level ($2n^2$).	5
Structure 1.3.6 (spectroscopic half)	HL ONLY	Recognise that the limit of convergence at higher frequency corresponds to ionization; calculate the ionization energy of hydrogen from the convergence limit using $E = h\nu$.	6

DATA BOOKLET — WHAT IS GIVEN TO YOU

Sections 1 (constants: c , h , N_A), 2 ($E = h\nu$, $c = \lambda\nu$) and 3 (the electromagnetic spectrum with wavelength ranges and colours) are **all provided in the exam**. You do **not** memorise the colour-wavelength numbers. You **do** memorise the *logic* and the *descriptions*, because those are where the marks are.

1 Light, energy and the electromagnetic spectrum SL + HL

1.1 The three quantities and how they interlock

Electromagnetic (EM) radiation can be described as a wave, or as a stream of particles of light called **photons**. Both pictures are needed in this topic.

Quantity	Symbol	SI unit	Meaning
Wavelength	λ (lambda)	metre, m (often nm)	Distance between two successive crests
Frequency	ν (nu)	hertz, Hz ($= s^{-1}$)	Number of waves passing a point per second
Energy of one photon	E	joule, J	The energy carried by a single quantum of light

$$c = \lambda\nu \quad E = h\nu \quad \text{therefore} \quad E = hc / \lambda$$

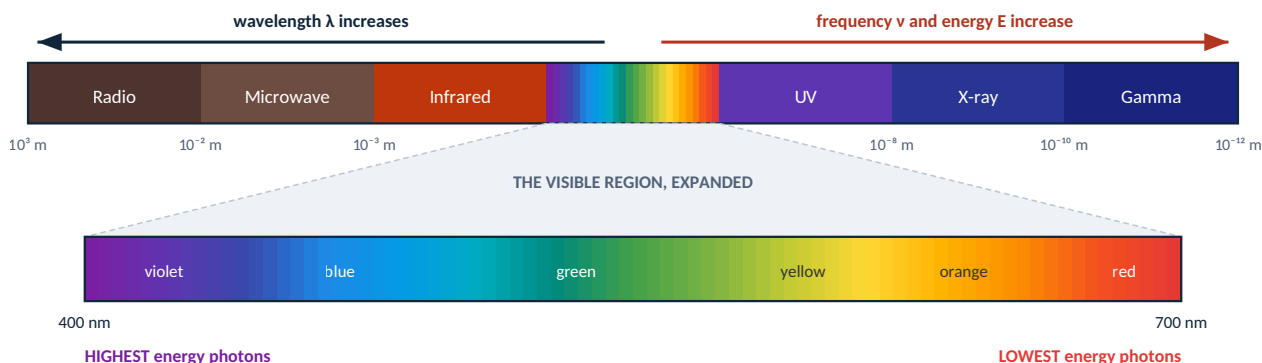
$$c = 3.00 \times 10^8 \text{ m s}^{-1} \text{ (speed of light)} \quad h = 6.63 \times 10^{-34} \text{ J s (Planck's constant)}$$

MEMORISE THIS SINGLE SENTENCE — IT ANSWERS HALF THE QUESTIONS IN THIS TOPIC

Short wavelength → high frequency → high energy. (λ is *inversely* proportional to both ν and E ; ν and E are *directly* proportional to each other.)

Across the visible region: **violet is the most energetic, red the least** — remember it as "*violet is violent*".

FIGURE 1 • THE ELECTROMAGNETIC SPECTRUM



The visible region is only a very narrow slice of the whole spectrum (400–700 nm).

Wavelength ranges are given in Section 3 of the data booklet — you are examined on the trend, not the numbers.

CLASSIC EXAM ERROR

Students write "red light has a higher wavelength *and* a higher energy". Wrong — red light has the **longest wavelength** and therefore the **lowest frequency and lowest energy** of visible light. If your two statements are not inverses of one another, you have made an error.

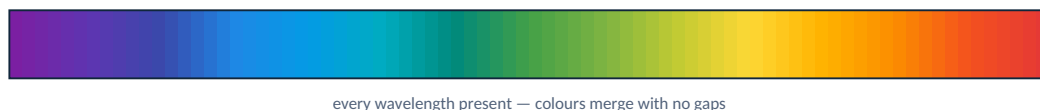
2 Continuous vs line spectra SL + HL

2.1 The two types

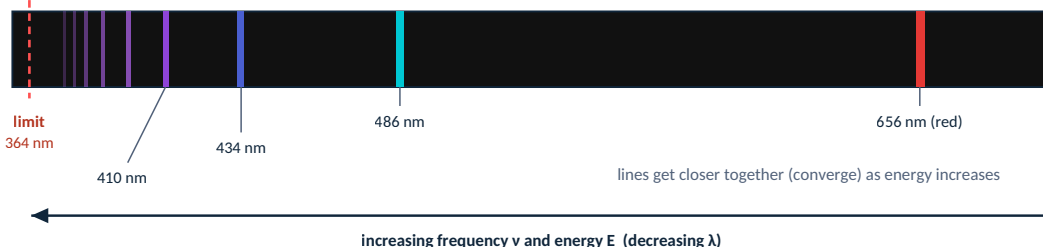
Feature	Continuous spectrum	Line (emission) spectrum
Appearance	An unbroken band in which each colour merges smoothly into the next	A series of discrete, separate coloured lines on a black background
Wavelengths present	All wavelengths within the range	Only certain specific wavelengths
Typical source	White light: sunlight, a hot tungsten filament (any hot solid)	Excited gaseous atoms : a hydrogen discharge tube, a flame test, a sodium street lamp
What it tells us	Energy can be emitted in any amount	Energy is emitted only in fixed amounts (quanta) → electrons occupy discrete energy levels
Identification	Not element-specific	Unique to each element — an atomic "fingerprint"

FIGURE 2 • CONTINUOUS SPECTRUM VS THE LINE EMISSION SPECTRUM OF HYDROGEN

(a) CONTINUOUS SPECTRUM — white light through a prism



(b) LINE EMISSION SPECTRUM — excited hydrogen atoms (visible / Balmer region)



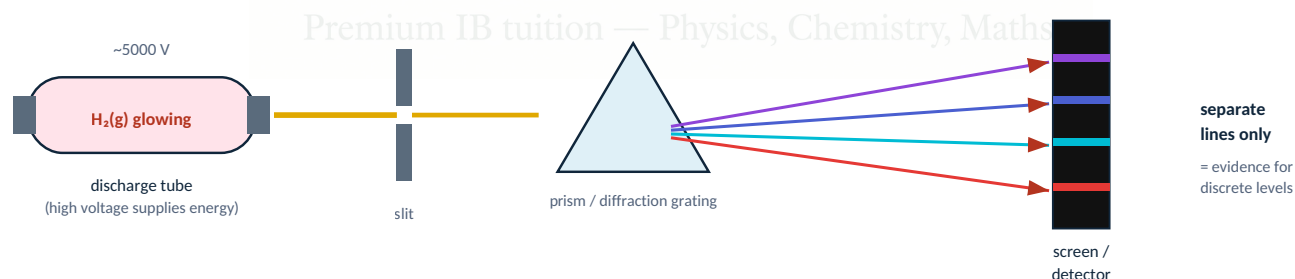
Note the direction of the axis: in an emission spectrum, energy increases towards the violet/UV end, and that is the end where the lines converge.

2.2 How an emission spectrum is produced — the four-step story

This is a very common "explain" question. Learn it as four numbered steps and write all four.

1. **Energy is supplied** to the gaseous atoms (by heat or, more usually, a high-voltage electrical discharge).
2. An electron **absorbs** exactly the right quantum of energy and is **promoted** (excited) from a lower to a higher energy level. The atom is now in an **excited state**.
3. The excited state is unstable, so the electron **falls back** to a lower energy level.
4. The energy difference is **emitted as a photon** of light of a specific frequency, given by $\Delta E = h\nu$. Because only certain ΔE values exist, only certain frequencies appear → a **line spectrum**.

FIGURE 3 • PRODUCING AND OBSERVING A LINE EMISSION SPECTRUM



KEY DEFINITION — LEARN WORD FOR WORD

A **quantum** is a fixed, discrete amount of energy. Energy levels in an atom are **quantised**: an electron may occupy only certain allowed energy levels and can never exist between them.

3 The hydrogen emission spectrum SL + HL

3.1 Why the lines converge — the single most examined idea

The energy levels of the hydrogen atom are **not evenly spaced**. As n increases, successive levels get **closer and closer together** and eventually converge at $n = \infty$. Because the energy *gaps* converge, the emitted photon energies converge, so the **spectral lines converge at the high-energy (high-frequency, short-wavelength) end**.

THREE-LINE ANSWER FOR "EXPLAIN THE CONVERGENCE"

1. Lines converge at higher frequency / shorter wavelength.
2. Because the **energy levels themselves converge** as n increases.
3. So the **differences in energy (ΔE) between successive levels become smaller**, and since $\Delta E = h\nu$, the frequencies of successive lines become closer together.

3.2 The three named series

Each series is defined by the level the electron **falls to**. This is the crucial link the syllabus asks for.

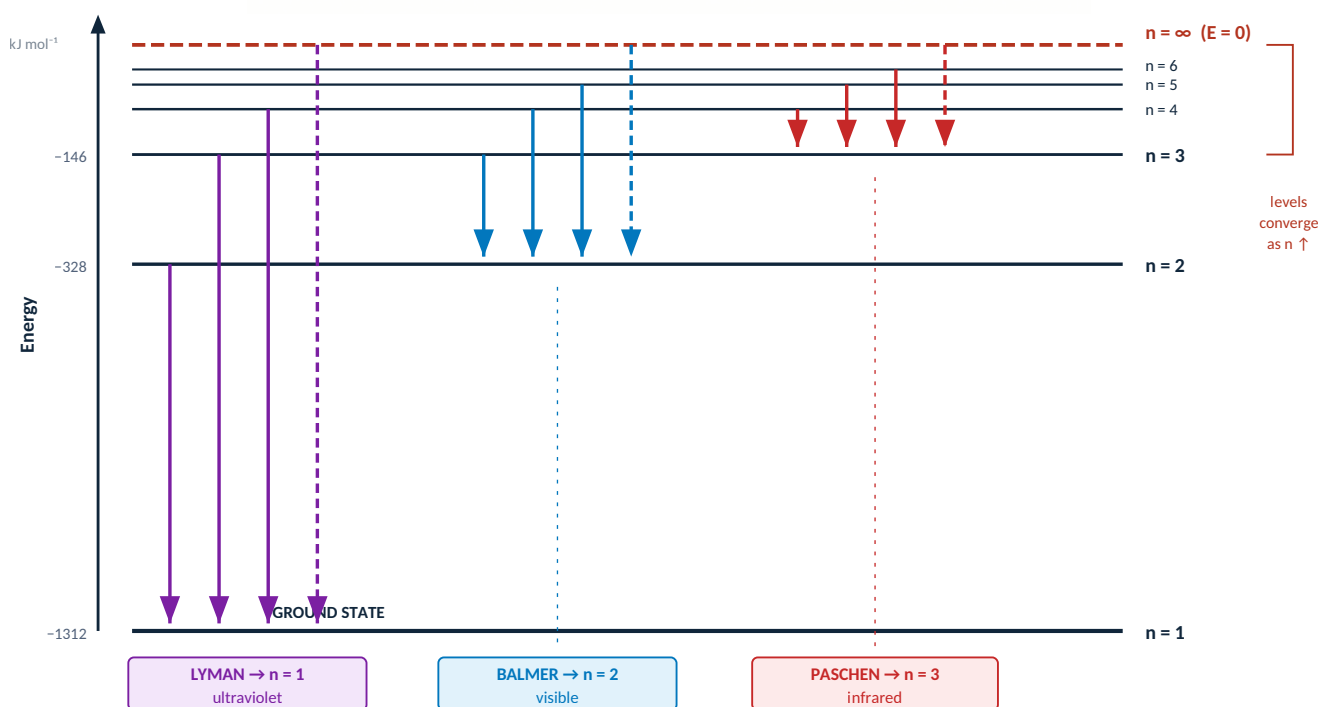
Series	Electron falls TO	Region of EM spectrum	Relative ΔE	Why
Lyman	$n = 1$ (from $n = 2, 3, 4$...)	Ultraviolet	Largest	The $n = 1$ level is far below all others, so the drop releases a lot of energy
Balmer	$n = 2$ (from $n = 3, 4, 5$...)	Visible	Intermediate	The only series we can see with the eye — 4 clear lines
Paschen	$n = 3$ (from $n = 4, 5, 6$...)	Infrared	Smallest	The upper levels are close together, so only small ΔE is released

MNEMONIC

L-B-P = 1-2 -3 = UV - Visible - IR. "Lyman Lands on level 1 and is the Largest jump (UV)." Then simply count down in energy.

You are **not** required to name the Brackett/Pfund series ($n = 4, 5$) — but you should know the pattern continues into the far IR.

FIGURE 4 • ENERGY LEVEL DIAGRAM FOR THE HYDROGEN ATOM, SHOWING THE THREE SERIES



Vertical spacing is drawn approximately to scale ($E_n \propto -1/n^2$). Dashed arrows are the transitions from $n = \infty$, which give the **convergence limit** of each series. Note how much larger the $n = 1 \rightarrow n = 2$ gap is than every other gap — that is why Lyman transitions land in the ultraviolet.

EXAM TECHNIQUE — DRAWING THIS DIAGRAM

If asked to sketch the levels, you get marks for: (i) horizontal lines labelled $n = 1, 2, 3, \dots$; (ii) lines **getting closer together** as n increases; (iii) **downward** arrows for emission; (iv) arrows ending on the correct level for the named series. Draw the $n = 1$ to $n = 2$ gap much bigger than all the others.

3.3 Reading the diagram: which transition gives which line?

- The **longest arrow** = biggest ΔE = highest frequency = shortest wavelength.
- Within the Balmer series, $3 \rightarrow 2$ is the **smallest** jump, so it gives the **lowest energy visible line = red, 656 nm**. The $6 \rightarrow 2$ jump is the largest, giving **violet, 410 nm**.
- Every Lyman line has more energy than every Balmer line, which has more than every Paschen line.

WORKED EXAMPLE 1 — IDENTIFYING A TRANSITION

Q. A line in the hydrogen spectrum has a wavelength of 486 nm. State the transition responsible and explain your reasoning.

[2]

A. 486 nm lies in the visible region, so the line belongs to the **Balmer series** and the electron must fall **to $n = 2$** . It is the second line from the red end, so it comes from $n = 4$. Transition: **$n = 4 \rightarrow n = 2$** .

4 Absorption spectra — the mirror image SL + HL

If white light is passed *through* a sample of cool gaseous atoms, the atoms absorb exactly the wavelengths they would otherwise emit. The result is a continuous spectrum crossed by **black lines** at exactly the same positions as the coloured emission lines.

- **Emission:** electron falls to a lower level, photon is **released** $\rightarrow$ coloured lines on black.
- **Absorption:** electron is promoted to a higher level, photon is **removed** $\rightarrow$ black lines on a coloured background.
- The identical line positions confirm the same set of quantised energy levels.

5 Main energy levels and the $2n^2$ rule SL + HL

Each main (principal) energy level is labelled with an integer $n = 1, 2, 3, 4, \dots$. The maximum number of electrons it can hold is:

$$\text{maximum number of electrons} = 2n^2$$

Main level n	$2n^2$	Max electrons	Sublevels available
1	2×1^2	2	1s
2	2×2^2	8	2s, 2p
3	2×3^2	18	3s, 3p, 3d
4	2×4^2	32	4s, 4p, 4d, 4f

DO NOT CONFUSE

$2n^2$ is the **capacity** of a level, not the number of electrons actually present. Potassium ($Z = 19$) has capacity 18 in level 3 but its third level holds only 8 electrons in the ground state, because the 4s sublevel is lower in energy than 3d. This is developed fully in the 1.3.2 notes.

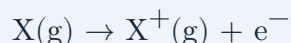
6 The convergence limit and ionization energy HL ONLY

6.1 The idea

As $n \rightarrow \infty$, the electron is infinitely far from the nucleus and is no longer part of the atom — the atom has been **ionized**. The very last line of a series, at the point where the lines merge, corresponds to the transition $n = \infty \rightarrow n = 1$. This is called the **convergence limit** (or series limit) and its frequency is the **highest** frequency in the whole spectrum.

DEFINITION — FIRST IONIZATION ENERGY

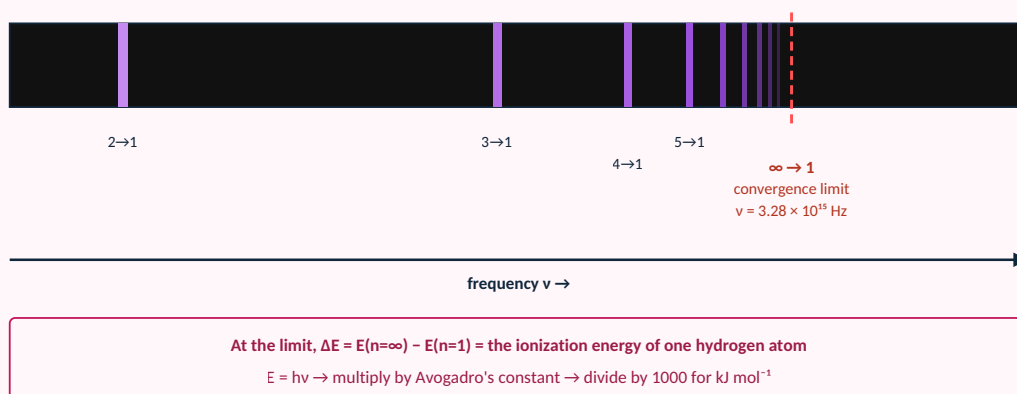
The **first ionization energy** is the minimum energy required to remove one mole of electrons from one mole of gaseous atoms in their ground state.



The three details examiners look for: **one mole, gaseous, ground state**.

Critical point: ionization energy is measured from the **ground state**, so only the **Lyman** convergence limit ($\infty \rightarrow 1$) gives the ionization energy of hydrogen. The Balmer limit ($\infty \rightarrow 2$) would give the energy to ionize an already-excited atom — a favourite trap in Paper 2.

FIGURE 5 • CONVERGENCE OF THE LYMAN SERIES TO THE IONIZATION LIMIT



6.2 The calculation — a fixed three-step routine

Step 1: $E = h\nu$ (or $E = hc/\lambda$ if given λ)

Step 2: multiply by $N_A = 6.02 \times 10^{23} \text{ mol}^{-1} \rightarrow \text{J mol}^{-1}$

Step 3: divide by 1000 $\rightarrow \text{kJ mol}^{-1}$

WORKED EXAMPLE 2 — IONIZATION ENERGY OF HYDROGEN FROM THE CONVERGENCE LIMIT

Q. The convergence limit of the Lyman series occurs at a frequency of $3.28 \times 10^{15} \text{ Hz}$. Calculate the first ionization energy of hydrogen in kJ mol^{-1} . [3]

Step 1 — energy of one photon

$$E = h\nu = (6.63 \times 10^{-34} \text{ J s})(3.28 \times 10^{15} \text{ s}^{-1}) = 2.17 \times 10^{-18} \text{ J}$$

Step 2 — scale to one mole

$$E = (2.17 \times 10^{-18})(6.02 \times 10^{23}) = 1.31 \times 10^6 \text{ J mol}^{-1}$$

Step 3 — convert

$$\text{IE} = 1.31 \times 10^6 / 1000 = 1.31 \times 10^3 \text{ kJ mol}^{-1} \text{ (1310 kJ mol}^{-1}\text{)}$$

Check against Section 8 of the data booklet: the accepted value for hydrogen is 1312 kJ mol^{-1} . This agreement is itself strong evidence that the model of quantised energy levels is correct.

WORKED EXAMPLE 3 — STARTING FROM A WAVELENGTH

Q. The Lyman convergence limit for hydrogen is at $\lambda = 91.2 \text{ nm}$. Calculate the first ionization energy of hydrogen. [3]

$$\lambda = 91.2 \text{ nm} = 91.2 \times 10^{-9} \text{ m} = 9.12 \times 10^{-8} \text{ m}$$

$$E = hc/\lambda = (6.63 \times 10^{-34} \times 3.00 \times 10^8) / (9.12 \times 10^{-8}) = 2.18 \times 10^{-18} \text{ J}$$

$$\times 6.02 \times 10^{23} = 1.31 \times 10^6 \text{ J mol}^{-1} = \mathbf{1.31 \times 10^3 \text{ kJ mol}^{-1}}$$

Unit trap: nm $\rightarrow$ m means $\times 10^{-9}$. Forgetting this is the single most common lost mark in this calculation.

HL CHECKLIST BEFORE YOU WRITE THE FINAL ANSWER

- Did I use the **Lyman** ($n = 1$) limit, not the Balmer limit?
- Did I convert nm to m?
- Did I multiply by N_A ? (An answer of order 10^{-18} is per atom, not per mole.)
- Did I divide by 1000 for kJ?
- Is my final answer of the order of 10^3 kJ mol^{-1} ? Real first ionization energies run from roughly 380 to 2400 kJ mol^{-1} .

7 One-page summary SL + HL

If the question says...	Your answer must contain...
"Explain how an emission spectrum arises"	Energy absorbed $\rightarrow$ electron promoted to higher level (excited) $\rightarrow$ electron falls back $\rightarrow$ photon emitted with $E = h\nu$ equal to ΔE between the levels
"Explain what a line spectrum tells us about the atom"	Only certain frequencies are emitted $\rightarrow$ only certain ΔE values exist $\rightarrow$ electrons occupy discrete / quantised energy levels
"Explain the convergence"	Energy levels converge as n increases $\rightarrow \Delta E$ between successive levels decreases $\rightarrow$ lines converge at high frequency / short wavelength
"Compare Lyman and Balmer"	Lyman: to $n = 1$, UV, higher energy. Balmer: to $n = 2$, visible, lower energy
"Which line has the greatest energy?"	The one with the shortest wavelength / highest frequency / largest ΔE (nearest the convergence limit)
HL "Calculate IE from the spectrum"	$E = h\nu$ (or hc/λ) $\rightarrow \times N_A \rightarrow \div 1000$; use the Lyman limit

8 Practice questions

Attempt these under timed conditions: roughly 1.5 minutes per mark. Full answers with mark-scheme wording follow in Section 9.

Part A — Multiple choice SL + HL

- Which of the following has the greatest energy per photon?
A. Red light B. Infrared C. Microwaves D. Ultraviolet
- Which statement about the emission spectrum of hydrogen is correct?
A. The lines converge at longer wavelength.
B. The lines in the visible region result from transitions to $n = 1$.
C. The lines converge at higher frequency.
D. The spectrum is continuous because hydrogen has only one electron.
- A line spectrum provides evidence that:
A. atoms contain neutrons. B. electron energy levels are discrete.
C. electrons have mass. D. light travels as a wave only.
- Which transition in a hydrogen atom emits radiation of the longest wavelength?
A. $n = 2 \rightarrow 1$ B. $n = 3 \rightarrow 1$ C. $n = 4 \rightarrow 3$ D. $n = 4 \rightarrow 2$
- The maximum number of electrons that can occupy the fourth main energy level is:
A. 8 B. 18 C. 32 D. 50
- HL ONLY Which quantity is obtained directly from the convergence limit of the Lyman series?
A. The second ionization energy of hydrogen. B. The first ionization energy of hydrogen.
C. The energy of the $n = 2$ level. D. The electron affinity of hydrogen.

Part B — Structured questions SL + HL

- Distinguish between a continuous spectrum and a line emission spectrum. [2]
- Explain, in terms of electron transitions, how the line emission spectrum of hydrogen is produced. [3]
- The lines in the emission spectrum of hydrogen converge at high frequency. Explain this observation. [3]
- The visible lines in the hydrogen spectrum occur at 656 nm, 486 nm, 434 nm and 410 nm. [3]
(a) State the name of this series and the energy level to which the electrons fall. [1]
(b) Identify the line produced by the $n = 3 \rightarrow n = 2$ transition and justify your choice. [2]
- Sketch an energy level diagram for the hydrogen atom for $n = 1$ to $n = 5$. On your diagram, label the levels and draw arrows to show one transition in the Lyman series and one transition in the Balmer series. State the region of the electromagnetic spectrum in which each appears. [4]
- Radiation of wavelength 434 nm is emitted by an excited hydrogen atom. Explain why radiation emitted in the Paschen series [2] has a longer wavelength than this.
- State the maximum number of electrons in the second and third main energy levels, and show how these values are obtained. [2]

Part C — HL extension HL ONLY

- The convergence limit in the Lyman series of hydrogen occurs at a frequency of 3.28×10^{15} Hz. Calculate the first ionization energy of hydrogen, in kJ mol^{-1} . [3]
- The convergence limit in the emission spectrum of a hydrogen atom occurs at $\lambda = 91.2$ nm. [4]
(a) Explain why this wavelength corresponds to ionization. [2]
(b) Calculate the energy, in J, of one photon of this radiation. [2]
- A student calculates the ionization energy of hydrogen using the convergence limit of the **Balmer** series and obtains 328 kJ mol^{-1} . Explain why this value is not the first ionization energy of hydrogen, and state what it does represent. [3]

17. The first ionization energy of helium is 2372 kJ mol^{-1} . Calculate the frequency of radiation, in Hz, corresponding to the convergence limit for helium.

[3]

The logo features the text 'iBLaurel' in a large, elegant serif font. To the right of the text is a stylized laurel leaf icon.

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9 Answers & mark-scheme notes

Part A

1. **D** — UV has the shortest wavelength of the four, so the highest frequency and, from $E = h\nu$, the greatest photon energy.
2. **C** — A is the reverse; B describes the Lyman (UV) series, not the visible one; D is false — hydrogen gives a line spectrum precisely *because* its energy levels are discrete.
3. **B** — discrete lines $\Rightarrow$ only certain ΔE values $\Rightarrow$ quantised levels.
4. **C** — longest wavelength = smallest ΔE . The $n = 4$ and $n = 3$ levels are the closest together of the pairs offered (Paschen, infrared).
5. **C** — $2n^2 = 2 \times 4^2 = 32$.
6. **B** — the Lyman limit is the $\infty \rightarrow 1$ transition, i.e. removal of the electron from the **ground state**, which is the definition of the first ionization energy.

Part B

7. [2]

- Continuous: all wavelengths/frequencies present, colours merge with no gaps. [1]
- Line: only certain discrete wavelengths/frequencies present, appearing as separate lines. [1]

Marker's note: the word "discrete" (or "separate", "specific") is required. "Lots of colours" scores zero.

8. [3]

- Electrons absorb energy and are promoted to a higher energy level (excited state). [1]
- Electrons fall back / return to a lower energy level. [1]
- Energy is emitted as a photon whose energy equals the difference between the levels, $\Delta E = h\nu$; only certain ΔE values are possible so only certain frequencies appear. [1]

9. [3]

- The energy levels converge / become closer together as n increases. [1]
- So the energy difference (ΔE) between successive levels decreases. [1]
- Since $\Delta E = h\nu$, the frequencies of successive lines become closer together, i.e. the lines converge at high frequency. [1]

10. [3]

(a) The **Balmer** series; electrons fall to $n = 2$. [1]

(b) The **656 nm** line. [1] The $3 \rightarrow 2$ transition is the **smallest** energy jump in the series, and smallest energy corresponds to the **longest wavelength**. [1]

11. [4]

- Five horizontal lines labelled $n = 1$ to $n = 5$, drawn **converging** (spacing decreasing) as n increases. [1]
- A **downward** arrow ending on $n = 1$ (e.g. $3 \rightarrow 1$), labelled Lyman. [1]
- A **downward** arrow ending on $n = 2$ (e.g. $4 \rightarrow 2$), labelled Balmer. [1]
- Lyman = **ultraviolet**; Balmer = **visible**. [1]

Marker's note: upward arrows describe absorption and lose the mark. Evenly spaced levels also lose a mark.

12. [2]

- Paschen transitions end at $n = 3$, and the levels above $n = 3$ are close together, so ΔE is smaller than for a Balmer transition. [1]
- Smaller $\Delta E \rightarrow$ lower frequency $\rightarrow$ longer wavelength ($\lambda = hc/E$). [1]

13. [2]

- $n = 2$: $2 \times 2^2 = 8$ electrons. [1]
- $n = 3$: $2 \times 3^2 = 18$ electrons. [1]

Part C **HL ONLY**

14. [3]

$$E = h\nu = 6.63 \times 10^{-34} \times 3.28 \times 10^{15} = 2.17 \times 10^{-18} \text{ J} \text{ [1]}$$

$$\times N_A: 2.17 \times 10^{-18} \times 6.02 \times 10^{23} = 1.31 \times 10^6 \text{ J mol}^{-1} \text{ [1]}$$

$$= 1.31 \times 10^3 \text{ kJ mol}^{-1} \text{ [1]}$$

Accept 1310–1320 kJ mol⁻¹. An answer left in J mol⁻¹ loses the final mark only.

15. [4]

(a) • It is the convergence limit, corresponding to the transition from $n = \infty$ to $n = 1$. [1]

- At $n = \infty$ the electron has been completely removed from the atom, i.e. the atom is ionized from its ground state. [1]

(b) $\lambda = 91.2 \times 10^{-9} \text{ m}$; $E = hc/\lambda = (6.63 \times 10^{-34} \times 3.00 \times 10^8) / (9.12 \times 10^{-8})$ [1]

$$= 2.18 \times 10^{-18} \text{ J} \text{ [1]}$$

16. [3]

- The Balmer convergence limit corresponds to the transition $n = \infty \rightarrow n = 2$, not $n = \infty \rightarrow n = 1$. [1]
- The first ionization energy must be measured from the **ground state** ($n = 1$), so the Lyman limit is required. [1]
- The value 328 kJ mol⁻¹ is the energy needed to ionize hydrogen atoms already **excited to the $n = 2$ level**. [1]

17. [3]

$$\text{Per atom: } E = (2372 \times 10^3) / (6.02 \times 10^{23}) = 3.94 \times 10^{-18} \text{ J} \text{ [1]}$$

$$\nu = E/h = 3.94 \times 10^{-18} / 6.63 \times 10^{-34} \text{ [1]}$$

$$= 5.94 \times 10^{15} \text{ Hz} \text{ [1]}$$

Sense check: helium's IE is larger than hydrogen's, so its convergence limit must be at a **higher** frequency than $3.28 \times 10^{15} \text{ Hz}$. It is.

End of Structure 1.3.1 • Next: **Structure 1.3.2 — Electron Configurations** (sublevels, orbitals, Aufbau/Hund/Pauli, ions, and the HL ionization-energy trends)

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